Problem Set 1

1. a) ∃x A(x)  
 b) ∀x [ (T(x) ⋀ S(x)) → A(x) ]  
 c) ­­⇁ ∀x [ T(x) → ⇁ A(x)]  
 d) x ≥ 3 ( T(x) ⋀ ⇁ S(x) )

2.

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| --- | --- | --- | --- | --- | --- | --- |
| **P** | **Q** | **R** | **⇁ (P ∨ ( Q ⋀ R ) )** | **( ⇁ P ) ⋀ (⇁ Q ∨ ⇁ R )** | **⇁ ( P ⋀ ( Q ∨ R ) )** | **⇁ P ∨ ( ⇁ Q ∨ ⇁ R )** |
| F | F | F | T | T | T | T |
| F | F | T | T | T | T | T |
| F | T | F | T | T | T | T |
| F | T | T | F | F | T | T |
| T | F | F | F | F | T | T |
| T | F | T | F | F | F | T |
| T | T | F | F | F | F | T |
| T | T | T | F | F | F | T |

a) True  
b) False

3. a)  
 (i) A ⋀ B = ⇁ (A nand B)  
 (ii) A ∨ B = (⇁ A) nand (⇁ B)  
 (iii) A ⇒ B = ((A nand B) nand B)

b) ⇁ A = (A nand A)

c) (1+1=3) nand A always evaluates to true  
 [((1+1=2) nand A) nand A] nand [((1+1=2) nand A) nand A] is always false.

4. Divide the 12 coins into two groups of 6. Weigh both groups, the heavier group can be set aside as it will not have the fake coin. Now divide the remaining 6 coins into two groups of 3 and weigh. The heavier group can be set aside. Now hold onto one coin while comparing the weights of the other two. If both those coins weigh the same, then the coin you are holding onto is fake. If one coin is lighter than the other, then that one is the fake.

5.